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INSTANTANEOUS ENERGY OPERATORS: APPLICATIONS TO SPEECH PROCESSING AND COMMUNICATIONS

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ABSTRACT

The nonlinear energy operator $\Psi(x) \equiv [\dot{x}]^2 - x\ddot{x}$ and its discrete-time counterpart have found numerous applications including development of the energy separation algorithm (ESA) for demodulating AM-FM signals, tracking speech modulations, and detecting various events in nonstationary signals. In this paper we first present some improvements on the energy operator and ESA when applied to demodulating speech resonances and using the extracted information signals for speech processing applications. Then we introduce some new nonlinear operators (differential in continuous time and quadratic in discrete time) that can provide higher-order energy measurements with applications to co-channel demodulation and separation of AM–FM signal mixtures. Finally, we present a cross-coupled ESA for co-channel demodulation.

The nonlinear continuous-time energy operator

$$\Psi(x)(t) \equiv [\dot{x}(t)]^2 - x(t)\ddot{x}(t)$$

and its discrete-time counterpart

$$\Psi(x)[n] \equiv x^2[n] - x[n-1]x[n+1]$$

were developed by Teager in his work on nonlinear speech modeling [1] and were both introduced systematically by Kaiser [2, 3]. Since its introduction, the energy operator has found numerous applications to demodulating AM and/or FM signals, tracking speech modulations, detecting various events in nonstationary signals and multiband analysis [5, 6, 7, 8, 9]. For example, given an AM–FM signal $x(t) = a(t)\cos(\int_0^t \omega(\tau)d\tau)$, by applying Ψ to both the signal and its derivative and separating their energies into amplitude and frequency components, Maragos, Kaiser and Quatieri [6, 8] have developed the following *energy separation algorithm (ESA)*

$$\omega(t) \approx \sqrt{\Psi(\dot{x})/\Psi(x)} \ , \ \ |a(t)| \approx \Psi(x)/\sqrt{\Psi(\dot{x})}$$

which can estimate the time-varying instantaneous frequency $\omega(t)$ and amplitude envelope $|a(t)|$ at any time instant, with negligible estimation error provided the amplitude and frequency signal do not vary too fast or too much with respect to the carrier.

In this paper we report (i) some improvements on the energy operator and ESA when applied to demodulating speech resonances and using the extracted information signals for a vocoder and speech recognition; and (ii) some new nonlinear operators (differential in continuous time and quadratic in discrete time) that can provide higher-order energy measurements with applications to co-channel demodulation and separation of AM–FM signal mixtures. Finally, we present a cross-coupled ESA for co-channel demodulation.

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1. SPEECH PROCESSING APPLICATIONS

In [5, 8] an AM-FM modulation model is introduced, that represents a speech resonance (formant) as an AM–FM signal. Each resonance is demodulated into instantaneous amplitude and frequency signals using the ESA. This modeling/analysis approach has been recently applied to formant tracking and speech coding [10], and, currently, to speech recognition.

First we introduce the *multiband demodulation formant tracking algorithm.* Filtering is performed by a bank of Gabor bandpass filters to isolate each speech resonance from the spectrum. Next, the amplitude envelope and instantaneous frequency are estimated for each band using the ESA. Short-time formant frequency and bandwidth estimates are obtained from the instantaneous amplitude and frequency signals. A simple decision algorithm determines the formant locations and bandwidths. Formant frequency and bandwidth (error bars) tracks for the sentence "Show me non-stop from Dallas to Atlanta" are shown in Fig. 1.

The *AM–FM modulation vocoder* extracts three or four time-varying formant bands from the spectrum by filtering the speech signal along the formant tracks (obtained as described above). The formant bands are demodulated into amplitude envelope and instantaneous frequency, which are decimated and coded. To synthesize the signal, the formant bands are reconstructed from the amplitude and phase signals, and added together.

Finally, one can obtain a non-parametric smooth spectral envelope from a multi-band filtering scheme by applying the energy operator Ψ on each band and taking the short-time average. This yields an *energy spectrum*, whose features we have used for speech recognition.

2. HIGHER-ORDER ENERGY OPERATORS

Instantaneous differences in the relative rate of change between two signals x, y can be measured via their Lie bracket

$$L[y, x] \equiv \dot{x}y - x\dot{y}$$

because $L[y, x]/xy = (\dot{x}/x) - (\dot{y}/y)$. Dots denote time derivatives. If $y = \dot{x}$, then $L[y, x]$ becomes the continuous-time Teager-Kaiser energy operator [2, 3]

$$\Psi(x) \equiv (\dot{x})^2 - x\ddot{x} = L[\dot{x}, x]$$

which has been used for tracking the energy of a source producing an oscillation [3, 2] and for signal and speech AM–FM demodulation [7, 8]. In the general case, if x and y represent displacements in some generalized motions, the

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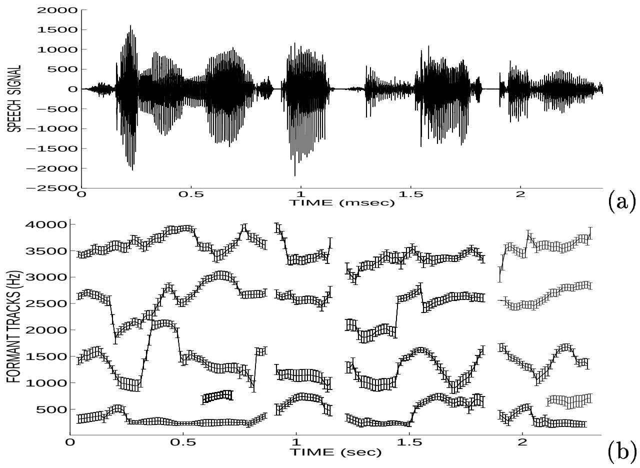

Figure 1: (a) Speech Signal (b) Formant tracks

quantity $L[\dot{y}, x] = \dot{x}\dot{y} - x\ddot{y}$ has dimensions of energy (per unit mass), and hence may be viewed as a *'cross energy'* between x and y. This energy-like quantity $\dot{x}\dot{y} - x\ddot{y}$ was used in [3, 4] to analyze the output $\Psi(x+y)$ of the energy operator applied to a sum of two signals.

In our work we use the cross energy between a signal x and its higher-order derivatives to develop higher-order energy measurements. Specifically, we define the k^{th}-order *differential energy operator (DEO)*

$$\Upsilon_k(x) \equiv L[x^{(k-1)}, x] = \dot{x}x^{(k-1)} - x x^{(k)}, \quad k = 0, \pm 1, \pm 2, \ldots$$

as yielding the cross energy between a signal $x(t)$ and its $(k-1)^{\text{th}}$ derivative (or integral), where

$$x^{(k)}(t) \equiv \begin{cases} d^k x(t)/dt^k, & k \geq 1 \\ x(t), & k = 0 \\ \int_{-\infty}^{t} x^{(k+1)}(\tau)d\tau, & k \leq -1 \end{cases}$$

denotes a signal derivative for positive order k or an integral for k negative. Of practical current interest are the DEOs of positive orders. The second-order DEO Υ_2, measuring the energy of a harmonic oscillator producing a signal x, gives to Υ_k the name 'energy' since it is identical to the standard energy operator Ψ. The zeroth-order operator $\Upsilon_0(x) = \dot{x}\int x - x^2$ was recognized in [4] as the negative of the energy of the signal integral. The first-order DEO yields zero for any signal. Two new and useful energy measurements are given by the third- and fourth-order DEOs:

$$\Upsilon_3(x) \equiv \dot{x}\ddot{x} - x x^{(3)} \quad , \quad \Upsilon_4(x) \equiv \dot{x}x^{(3)} - x x^{(4)}$$

Note that (as also observed in [4])

$$\Upsilon_3(x) = d\Psi(x)/dt \quad , \quad \Upsilon_4(x) = d\Upsilon_3(x)/dt \; - \; \Psi(\dot{x})$$

Hence the third-order DEO Υ_3 is an *energy velocity* operator, whereas the fourth-order DEO Υ_4 has dimensions of *energy acceleration.* In general, the higher-order operators can be generated by lower-order operators with a 2-step recursion: $\Upsilon_k(x) = d\Upsilon_{k-1}(x)/dt \; - \; \Upsilon_{k-2}(\dot{x})$.

When the energy operators Υ_k are applied to a sine wave, they yield products of powers of the amplitude and frequency. This creates the energy recursion

$$E_k = -\omega^2 E_{k-2} \quad , \quad E_k = \Upsilon_k[A\cos(\omega t + \theta)],$$

with initial conditions $E_0 = -A^2$ and $E_1 = 0$. Running this recursive equation in both forward and backward order index k yields

$$\Upsilon_k[A\cos(\omega t + \theta)] = \begin{cases} 0, & k = \pm 1, \pm 3, \pm 5, \ldots \\ (-1)^{1+\frac{k}{2}} A^2 \omega^k, & k = 0, \pm 2, \pm 4, \ldots \end{cases}$$

If the amplitude A and/or frequency ω are slowly time-varying, i.e., if the input is an AM–FM signal, then the above energy equations are approximately valid provided that $A = A(t)$ and $\omega = \omega(t)$ do not vary too fast or too much with respect to the carrier frequency. Further, because $A^2\omega^k$ are low-pass signals, the above instantaneous energy measures can be used for robust estimation of amplitude and frequency information in time-varying sinusoids.

An application of the fourth-order DEO Υ_4, in conjunction with the standard energy operator $\Upsilon_2 \equiv \Psi$, is to estimate the amplitude and frequency of a (possibly time-varying) sinusoid $x(t) = A\cos(\omega t + \theta)$:

$$\omega = \sqrt{-\Upsilon_4(x)/\Upsilon_2(x)} \quad , \quad |A| = \Upsilon_2(x)/\sqrt{-\Upsilon_4(x)}$$

This is an energy separation algorithm, slightly different from the one in [8], which can also be used for AM-FM demodulation.

An application of the third-order DEO Υ_3 is to estimate the energy dissipation rate in damped oscillations. Namely, given a damped cosine, the damping factor can be found using Υ_3 and the energy operator. Thus, if $x(t) = Ae^{-rt}\cos(\omega t + \theta)$, $r > 0$, then

$$r = -\Upsilon_3(x)/2\Upsilon_2(x) = -\, 0.5 \;\; d(\log \Upsilon_2(x))/dt$$

Applying the energy operators to sampled signals requires replacing derivatives with differences. This leads to a variety of discrete energy operators for each order k. The simplest approach is to first discretize the cross signal operator $L[y, x]$ and then replace derivatives with time shifts. Namely, replacing continuous-time signals $x(t)$ with sequences $x_n = x(nT)$ of their samples, also denoted as $x[n]$, and first-order derivatives $\dot{x}(t)$ with backward differences $\Delta_b x[n] = (x[n] - x[n-1])/T$ converts the continuous-time operator $L[y, x](t)$ into the discrete-time operator

$$C(x[n], y[n]) \equiv x[n]y[n-1] - x[n-1]y[n]$$

where we henceforth assume $T = 1$. (Using symmetric differences $\Delta_s x[n] = (x[n+1] - x[n-1])/2$ to replace time derivatives yields a symmetric discrete operator equal to the average of C at two consecutive samples.) Using $y[n] = x[n+1]$ makes C identical to the discrete Teager-Kaiser energy operator [2]

$$\Psi(x[n]) \equiv x^2[n] - x[n-1]x[n+1] = C(x[n], x[n+1])$$

Generalizing the above result by using $y[n] = x[n+k]$ in C leads us to develop discrete-time higher-order energy measurements for a signal $x[n]$. For example, we define the k^{th}-order discrete[1] energy operator

$$\begin{aligned} \Upsilon_k(x[n]) \quad &\equiv \quad C(x[n], x[n+k-1]) \quad , \quad k = 0, 1, 2, 3, \ldots \\ &= \quad x[n]x[n+k-2] - x[n-1]x[n+k-1] \end{aligned}$$

For $k = 1$ we always get zero since $\Upsilon_1 \equiv 0$. For $k = 2$ we obtain the standard discrete energy operator $\Upsilon_2 \equiv \Psi$. For $k = 3$, we obtain an *asymmetric discrete energy velocity operator*

$$\Upsilon_3(x_n) \equiv x_n x_{n+1} - x_{n-1}x_{n+2}$$

[1]For simplicity the same symbol is used for both the continuous- and discrete-time operators Υ_k and Ψ.

whereas $k = 4$ yields a discrete energy acceleration operator:

$$\Upsilon_4(x_n) \equiv x_n x_{n+2} - x_{n-1}x_{n+3}$$

Important aspects of Υ_k are the length of its corresponding index window and its time alignment (a)symmetry. Next we investigate these issues for $k = 3$. Since Υ_3 requires a 4-sample moving window $[n-1, n+2]$, its output at the window's center occurs at the continuous time instant $t = (n + 0.5)T$. One simple approach to eliminate this time misalignment is to replace $\Upsilon_3(x_n)$ with its average over two consecutive samples and thus have a *symmetric* third-order energy operator

$$\Upsilon_{3s}(x_n) \equiv (\Upsilon_3(x_n) + \Upsilon_3(x_{n-1}))/2$$

with a 5-sample window $[n-2, n+2]$.

In [14] alternative approaches have been proposed to discretizing Υ_k, which require a small window and satisfy recursive formulas of the same type as their continuous counterparts. In general, the best type of discretization of higher-order energies depends on the specific application.

Applying the operators Υ_k to discrete damped cosines yields discrete energy equations

$$\Upsilon_k[Ar^n \cos(\Omega n + \theta)] = A^2 r^{2n+k-2} \sin(\Omega)\sin[(k-1)\Omega]$$

which are useful for parameter estimation in sinusoids. In addition, these energy equations also hold approximately when the cosine has time-varying amplitude and frequency that do not vary too fast or too much with respect to the carrier, i.e. when the input is a sampled AM–FM signal. This then allows to find discrete AM–FM demodulation algorithms by combining the above energy equations of various orders. For example, by using Υ_2, Υ_3, and the undamped cosine energy equations $\Upsilon_k[A\cos(\Omega n + \theta)] = A^2 \sin(\Omega)\sin[(k-1)\Omega)]$ for $k = 2, 3$, a discrete algorithm was found in [12] for tracking instantaneous frequencies, which is closely related to the discrete energy separation algorithm in [8].

We conclude by noting that, all the above discrete higher-order energy operators can be unified as special cases of a class of *quadratic energy operators* Q_{km}, or their weighted linear combinations, where

$$Q_{km}(x[n]) \equiv x[n]x[n+k] - x[n-m]x[n+k+m]$$

for $k = 0, 1, 2, ...,$ $m = 1, 2, ...$ These operators have also been studied independently by Kaiser [11]. The class Q contains all the discrete higher-order energy operators Υ_k since $Q_{k1} \equiv \Upsilon_{k+2}$; e.g., $Q_{01} \equiv \Psi$ and $Q_{11} \equiv \Upsilon_3$. For $k = 0$ the operators Q_{0m} can also be viewed as special cases of the class of quadratic detectors $\sum_m h_m x[n+m]x[n-m]$ proposed in [13]. The general operators Q_{km} satisfy the following energy equations:

$$Q_{km}[Ar^n \cos(\Omega n + \theta)] = A^2 r^{2n+k} \sin(m\Omega)\sin[(m+k)\Omega]$$

In addition, each Q_{km} can be generated recursively from other similar operators of lower orders k, m.

3. CO–CHANNEL DEMODULATION

We present a nonlinear algorithm for the demodulation of two-component AM–FM signals of the form

$$x(t) = a_1(t)\cos(\int_0^t \omega_1(\tau)d\tau) + a_2(t)\cos(\int_0^t \omega_2(\tau)d\tau)$$

using the generating equation of the mixture signal $x(t)$ and higher-order energy operators. First we exploit the structural properties of a mixture of two sinusoidal signals by treating the mixture signal as a solution to a generating differential or difference equation (GDE) [16]. The coefficients of the GDE are then expressed in terms of higher-order energy operators to facilitate simultaneous separation of a two-component AM–FM signal into components and demodulation of the components into instantaneous frequency and amplitude signals.

Consider a mixture of two sinusoidal signals

$$x(t) = a_1 \cos(\omega_1 t + \theta_1) + a_2 \cos(\omega_2 t + \theta_2)$$

which satisfies the following fourth-order GDE

$$x^{(4)} + c_1\ddot{x} + c_2 x = 0$$

where $c_1 = \omega_1^2 + \omega_2^2$ and $c_2 = \omega_1^2\omega_2^2$. Using the GDE and its derivative and solving the resulting 2×2 linear system of equations yields the following expressions for the coefficients:

$$c_1 = -\Upsilon_5(x)/\Upsilon_3(x) \quad , \quad c_2 = \Upsilon_3(\ddot{x})/\Upsilon_3(x)$$

and the following frequency estimation algorithm

$$\omega_{1,2} = \sqrt{0.5\,[c_1 \pm \sqrt{c_1^2 - 4c_2}\,]}$$

These frequency estimates are then used in conjunction with 2nd-order energy operators to develop estimates for the amplitude as follows:

$$a_{1,2}^2 = \frac{\omega_{2,1}^4(\Psi(x^{(3)}) - \omega_1^2\omega_2^2\Psi(\dot{x})) - \omega_{2,1}^6(\Psi(\ddot{x}) - \omega_1^2\omega_2^2\Psi(x))}{\omega_1^4\omega_2^4(\omega_1^2 - \omega_2^2)^2}$$

The proposed algorithm yields exact quantities for a mixture of two cosines. If the signal $x(t)$ is an AM–FM mixture where the message signals do not vary too fast or too much with respect to the carriers, then the algorithm yields efficient estimates for the instantaneous amplitude and frequency signals of each component. We refer to the above two-component instantaneous frequency and amplitude estimation procedure as the *Energy Demodulation* of *Mixtures (EDM)*.

In **discrete-time** a mixture of two sinusoids

$$x_n = a_1 \cos(\Omega_1 n + \theta_1) + a_2 \cos(\Omega_2 n + \theta_2)$$

satisfies the fourth-order GDE

$$c_1(x_{n-1} + x_{n-3}) + c_2 x_{n-2} = -(x_n + x_{n-4})$$

with $c_1 = -2(\cos\Omega_1 + \cos\Omega_2)$, $c_2 = 4\cos\Omega_1\cos\Omega_2 + 2$. From the GDE at time instants n and $n+1$ we obtain

$$c_1 = \frac{\Upsilon_3(x_{n-3}) - \Upsilon_3(x_{n-1})}{\Psi(x_{n-1}) - \Psi(x_{n-2})}$$

$$c_2 = \frac{\Psi(x_n) - \Psi(x_{n-3})}{\Psi(x_{n-1}) - \Psi(x_{n-2})} + \frac{\Upsilon_4(x_{n-2}) - \Upsilon_4(x_{n-3})}{\Psi(x_{n-1}) - \Psi(x_{n-2})}$$

and the discrete-time frequency estimation formula

$$\Omega_{1,2} = cos^{-1}(0.25\,[-c_1 \pm \sqrt{c_1^2 - 4c_2 + 8}\,])$$

The instantaneous amplitude is obtained from the frequency estimates as

$$a_{1,2}^2 = \frac{S_{2,1}^4(\Psi({\Delta_s}^3 x) - S_1^2 S_2^2 \Psi(\Delta_s x))}{S_1^4 S_2^4 (S_2^2 - S_1^2)^2} - \frac{S_{2,1}^6(\Psi({\Delta_s}^2 x) - S_1^2 S_2^2 \Psi(x))}{S_1^4 S_2^4 (S_2^2 - S_1^2)^2}$$

where $S_{1,2} = \sin(\Omega_{1,2})$ and ${\Delta_s}^m x = \Delta_s({\Delta_s}^{m-1}x)$.
For a sum of two (slowly varying) AM–FM signals the EDM algorithm produces efficient instantaneous amplitude and frequency estimates; e.g., consider the sum of

two sinusoidally modulated and spectrally close AM–FM signals

$$
\begin{aligned}
x[n] &= \sum_{i=1}^{2} a_i[n] \cos\left(\int_0^n \Omega_i[m]dm\right) \\
\Omega_i[n] &= \Omega_{ci} + \Omega_{mi}\cos\left(\Omega_{fi} n + \theta_i\right) \\
a_i[n] &= 1 + \kappa_i \cos\left(\Omega_{ai} n + \beta_i\right) \quad , \quad i = 1,\ 2
\end{aligned}
$$

with parameters

$$
\Omega_{ci} = \tfrac{\pi}{4}, \tfrac{\pi}{4} + \tfrac{\pi}{50}, \quad \kappa_i = 0.08, 0.07, \quad \theta_i = 0, \pi
$$
$$
\beta_i = \tfrac{\pi}{2}, 0 \quad \Omega_{ai} = \Omega_{mi} = \Omega_{fi} = \tfrac{\Omega_{ci}}{150}.
$$

In Fig. 2(a)-(d) we show the smoothed (5-pt median followed by moving average filter) instantaneous amplitude and frequency estimates and the actual quantities. The coefficients signals c_1, c_2 are also median pre-filtered.

Overall, the EDM algorithm [15] is efficient because it exploits the natural symmetry of the signal mixture to estimate a minimal number of coefficients of the generating differential/difference equation and uses differential/quadratic energy operators that have a low complexity and excellent time resolution. It also has lower complexity and yields smaller estimation errors than the algorithm in [17] which uses operators based on the determinant of 4×4 Toeplitz matrices.

For the separation and demodulation of two component signals in additive white Gaussian noise (AWGN), we first filter the noisy mixture signal with a single bandpass filter, with center frequency set to the average carrier frequency of the two components and bandwidth set to the sum of the component bandwidths. The EDM algorithm is then applied on this filtered signal to produce amplitude and frequency estimates for each component.

An alternative scheme we developed for separation and demodulation of two-component FM signals in AWGN is the **cross-coupled ESA**: i.e., to use the ESA in a cross-coupled configuration in an iterative fashion. Initial signal separation is accomplished by bandpass filtering the multicomponent signal using two Gabor filters as in [9]. Separation for subsequent iterations is accomplished by subtracting an estimate of the second component from the mixture to obtain an estimate of the first component. The demodulation is accomplished by applying the ESA to the separated components. The instantaneous frequency and amplitude estimates are then used to resynthesize the components for the next iteration.

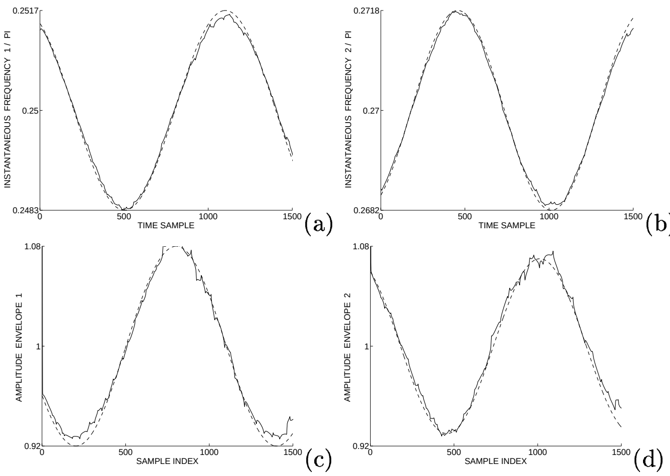

Figure 2: Estimates (solid lines) of $\Omega_{1,2}[n]$ (a),(b) and $a_{1,2}[n]$ (c),(d) for a two-component AM–FM mixture.

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